Addendum to: Area density of localization-entropy I

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January 2007

Abstract

This addendum adds some comments to a published paper on area density of localization-entropy. The setting of holography and localization-entropy is generalized to double cones and black holes with bifurcate Killing horizons.

PACS: 11.10.-z, 11.25.Hf, 04.70.Dy

1 A brief resumé of lightfront holography

The algebraic setting of QFT the *holographic projection*[1] on null-surfaces replaces the system of subalgebras of a given localized algebra (associated with bulk subregions) by a new system of subalgebras indexed with subregions of the causal horizon. In [1] this construction was presented for the holography onto the lightfront null-surface. It is precisely the relation of *localization* with *vacuum-fluctuation-caused thermal properties* and their mathematical connection with *modular operator algebra theory* which forces one to substitute the old incomplete (and not entirely correct) ideas around "lightcone quantization" with *lightfront holography* in the sense of this paper.

The starting point is the previously presented fact that a lightfront LF is the linear extension of the upper horizon ∂W of a fixed wedge W region (all regions are open) which reveals the thermal manifestations of vacuum polarization and permits to use them in a constructive way. The *causal shadow property* of the algebraic approach (stating that an operator algebra associated with a region $\mathcal{O}$ is equal to that associated with the generally larger causally completed region

[1] The terminology goes back to t'Hooft who coined it for a property which he expected to be an important attribute of a future theory of quantum gravity (QG). In the present context it refers to a property of localized quantum matter which is not a new requirement but rather follows from the standard causal locality and spectral properties of QFT. Although it has no direct relation with the still elusive QG, it explains some of the properties (e.g. the area proportionality of vacuum fluctuation caused entropy) which have been attributed to QG.

$\mathcal{O}''$) identifies the operator algebra belonging to the characteristic surface ∂W with that of its causally associated bulk region W

$$\mathcal{A}(\partial W) \equiv \mathcal{A}(W) \tag{1}$$

But it does not provide algebraic localization properties for subregions of these semi-global operator algebras. The aim of holography is to construct a coherent system of subalgebras $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\partial W)$ (where from now the letter $\mathcal{O}$ denotes finitely extended regions on LF) and this local refinement of the lightfront algebra $\mathcal{A}(LF)$ is the source of the computational power of lightfront holography versus the old “lightcone quantization“. In the spirit of Leibniz's identification of spacetime as an *ordering device for matter*, holography is a rather radical change of that device and of the physical interpretation, but it maintains the Hilbert space of the bulk theory as well as those subalgebras in the bulk which are causal shadows of regions on LF; these shadow regions consist of those wedges W whose upper horizons ∂W are halfplanes of LF.

The often raised question whether thermal manifestations of localization, in particular localization-entropy, are phenomena associated with W or ∂W is somewhat academic in view of the identity (1). It would be reasonable to maintain that from the viewpoint of the material substrate this distinction is not intrinsic but rather pendent on one's choice of the spacetime ordering device. Bulk regions (black holes) and their causal (event) horizons are mainly distinguished in this sense of Leibniz. Holography is a more radical form of the recent statement in the setting of QFT in CST [10] that local quantum physics in isometric regions of different universes is isomorphic. All these statements are special cases of the still somewhat mysterious possibility of encoding all material properties (including the distiction between different kinds of quantum matter) as well as the geometric spacetime positioning into appropriately defined modular inclusions and intersection of a finite number of copies of the same abstract operator algebra (a *monade,* or in the terminology of operator algebras a hyperfinite type III_1 von Neumann factor algebra). This may be viewed as an abstract ordering device in Hilbert space which generates the richness of the *material world and spacetime from the positioning of a finite number of monades.*

The symmetry group $\mathcal{G}$ of LF is known to be a 7-parametrig subgroup of the 10-parametric Poincaré group which is generated by of lightray translations, dilations (the LF projections of the W-preserving boost) and certain linear transformations of the coordinate directions in the transverse space $\mathbb{R}^2 \subset LF$ (resulting from the LF projection of the 3-parameter Wigner little group which leaves the lightray invariant) together with transverse translations. The local subalgebras $\mathcal{A}(\mathcal{O})$ with $\mathcal{O} \subset LF$ of the holographic projection are obtained by successive steps involving intersections of wedge algebras obtained from W by the application of $\mathcal{G}$. By iterating this process of taking intersections and acting with $\mathcal{G}$, one obtains a a coherent net of algebras $\mathcal{A}(\mathcal{O})$ indexed by arbitrarily small regions $\mathcal{O} \subset \ LF$.

The results in [1] may be summarized as follows

1. The system of LF subalgebras $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset LF}$ tensor-factorizes transversely with the vacuum being free of transverse entanglement

$$\begin{aligned} &\mathcal{A}(\mathcal{O}_1\cup\mathcal{O}_2)=\mathcal{A}(\mathcal{O}_1)\otimes\mathcal{A}(\mathcal{O}_2),\ (\mathcal{O}_1)_\perp\cap(\mathcal{O}_2)_\perp=\emptyset \\ &\langle\Omega\,|\mathcal{A}(\mathcal{O}_1)\otimes\mathcal{A}(\mathcal{O}_2)|\,\Omega\rangle=\langle\Omega\,|\mathcal{A}(\mathcal{O}_1)\,|\Omega\rangle\,\langle\Omega|\,\mathcal{A}(\mathcal{O}_2)|\,\Omega\rangle \end{aligned} \tag{2}$$

2. Extensive properties as entropy and energy on LF are proportional to the extension of the transverse area.

3. The area density of localization-entropy in the vacuum state for a system with sharp localization on LF diverges logarithmically

$$s_{loc}=\lim_{\varepsilon\to 0}\frac{c}{6}\,|ln\varepsilon|+... \tag{3}$$

   where ε is the size of the interval of "fuzziness" of the boundary in the lightray direction which one has to allow in order for the vacuum polarization cloud to attenuate and the proportionality constant c is (at least in examples) the central extension parameter of the Witt-Virasoro algebra.

The following comments about these results are helpful in order to appreciate some of the physical consequences as well as possibilities of extension to more general null-surfaces.

The transverse factorization with respect to the vacuum state is the consequence of a general structural theorem [2] of Local Quantum Physics (LQP). The latter states that two operator algebras $\mathcal{A}_i\subset B(H), i=1,2$ with $[A_1,U(a)\mathcal{A}_2U(a)^*]=0\ \forall a$, and $U(a)$ a translation with nonnegative generator which fulfills the cluster factorization property (i.e. asymptotic factorization in correlation functions for infinitely large cluster separations) with respect to a unique $U(a)$-invariant state vector Ω automatically satisfy the stronger tensor factorization property (a strong form of statistical independence) in the sense of the above statement **1** which in turn implies clustering and commutativity. In the case at hand the tensor factorization follows as soon as the regions have no transverse overlap[2].

Evidently this theorem has far-reaching consequences for algebraic nets indexed by subregions on null-surfaces in curved spacetime. Besides the quantum mechanical nature in transverse directions on LF it guaranties the absence of corresponding vacuum polarizations in transverse directions on the mantle of a lightcone or of a compact double cone in conformal QFTs for which these regions are obtained from wedges by conformal transformations. The asymptotic cluster factorization holds for spacelike separations and in a weaker form also for lightlike cluster separations in the ambient Minkowski spacetime bulk [3].

Let W be the x_0-x_3 wedge in Minkowski spacetime which is left invariant by the x_0-x_3 Lorentz-boosts. Consider a family of wedges W_a which are obtained by sliding the W along the $x_+=x_0+x_3$ lightray by a lightlike translation

[2] Locality in both directions shows that the lightlike translates $\langle\Omega\,|AU(a)B|\,\Omega\rangle$ are boundary values of entire functions and the cluster property together with Liouville's theorem gives the factorization.

$a > 0$ into itself. The set of spacetime points on LF consisting of those points on ∂W_a which are spacelike to the interior of W_b for $b > a$ is denoted by $\partial W_{a,b}$; it contains all points $x_+ \in (a, b)$ with an unlimited transverse part $x_\perp \in R^2$. These regions are two-sided transverse "slabs" on LF. To get to intersections of finite size one may “tilt” these slabs by the action of certain subgroups in $\mathcal{G}$ which change the transverse directions. Using the 2-parametric subgroup $\mathcal{G}_2$ of $\mathcal{G}$, which is the restriction to LF of the two “translations” in the Wigner little group (i.e. the subgroup fixing the lightray in LF), it is easy to see that this is achieved by forming intersections with G_2- transformed slabs $\partial W_{a,b}$

$$\partial W_{a,b} \cap g(\partial W_{a,b}),\ g \in \mathcal{G}_2 \tag{4}$$

Continuing with forming intersections and unions, one can get to finite convex regions $\mathcal{O}$ of a quite general shape.

An alternative method for obtaining holographically projected compactly localized subalgebras $\mathcal{A}(\mathcal{O}), \mathcal{O} \subset LF$ which does not make use of transverse symmetries, consists in intersecting $\mathcal{A}(\partial W_a)$ with suitable algebras in the bulk which are localized in a tubular neighborhood of $\mathcal{O}$ [4]. This is useful for null-surfaces in curved spacetime (next section).

The nontrivial question is now whether this geometric game can be backed up by the construction of a nontrivial net of operator algebras which are indexed by those regions. Since a subregion on ∂W, which either does not extend to infinity in the x_+ lightray direction or lacks the two-sided transverse extension does not cast any causal shadow[3], one cannot base the nontriviality of algebras $A(\partial W_{a,b})$ on the causal shadow property. If this algebra would be trivial (i.e. consist of multiples of the identity), the motivation for the use of holographic projections (which consists in obtaining a simpler description of certain properties) would fall flat and with it the dream of simplifying certain physical aspects via lightlike holography.

It has been customary in the algebraic approach to add structural properties concerning intersections to the “axiomatic” list of algebraic requirements if they can be derived in the absence of interactions and at least formulated in the presence of interaction-caused vacuum polarization clouds, remembering that the intrinsic characterizations of interactions is the inexorable presence of such clouds in local states (states by acting with a local operator onto the vacuum). This is a generalization to the setting of algebraic QFT of a theorem known since the early 60s that a pointlike field which is not free upon application to the vacuum generates necessarily a vacuum polarization cloud in addition to to a one-particle component. For noncompact localization regions as wedges there are vacuum **p**olarization-**f**ree **g**enerators even in the presence of interactions (the PFGs in [5]). There is presently no indication that the nontriviality of the algebraic intersections of bulk algebras needed for the construction of the local algebras $\mathcal{A}(\mathcal{O})$ on the lightfront is in any way impeded by interactions. But at

[3]In the classical setting this means that such characteristic data in contrast to data on ∂W (W arbitrary) on LF do not define a hyperbolic propagation problem in the ambient spacetime.

least there exists a proof [6] that within the family of two-dimensional factorizing models the double cone intersections of two wedge algebras in the bulk act cyclically on the vacuum. After 80 years of QFT without having been able to secure the existence of any nontrivial model (except superrenormalizable models with a finite wave-function renormalization), the setting of operator algebras, in particular modular theory, has opened new avenues beyond perturbation theory. The proof that the nontriviality continues to be valid for the intersections needed in the holographic projection is only a matter of time.

In the following the nontriviality of holography will be shown in the absence of interactions. It is well-known that the system of interaction-free localized operator algebras $\mathcal{A}(\mathcal{O}), \mathcal{O} \subset LF$ (which are constructed with the help of Weyl algebra generators from free fields) do indeed pass this nontriviality test for sufficiently many finite regions $\mathcal{O}$ of interests [7]. In fact these algebras on LF have also pointlike generators; the generating fields are the well-known "lightcone quantization" (or "$p \to \infty$ frame" method) fields A_{LF} (using the abbreviation $x_{\pm} = x^0 \pm x^3,\ p_{\pm} = p^0 + p^3 \simeq e^{\mp\theta},\ \theta$ the rapidity):

$$A_{LF}(x_+, x_\perp) \simeq \int \left(e^{i(p_-(\theta)x_+ + ip_\perp x_\perp} a^*(\theta, p_\perp) d\theta dp_\perp + h.c. \right) \tag{5}$$

$$\left\langle \partial_{x_+} A_{LF}(x_+, x_\perp) \partial_{x\prime_+} A_{LF}(x'_+, x'_\perp) \right\rangle \simeq \frac{1}{\left(x_+ - x'_+ + i\varepsilon\right)^2} \cdot \delta(x_\perp - x'_\perp)$$

$$\left[\partial_{x_+} A_{LF}(x_+, x_\perp), \partial_{x\prime_+} A_{LF}(x'_+, x'_\perp)\right] \simeq \delta'(x_+ - x'_+)\delta(x_+ - x'_\perp)$$

where, we for the sake of brevity and structural clarity, we left out all unimportant constant and took the lightlike derivative in order to avoid technicalities about logarithmic zero mass correlations (a pseudo problem, resolved in terms of test-functions as well-known from the massless field in 2-dim. QFT[4]). *The only physical purpose of this auxiliary field is to generate a system of local operator algebras on LF*; their correlation function have no other direct relation to the physical correlation in the bulk[5]; in particular in case of free fields the A_{LF} correlation functions on LF and the bulk correlation in terms of A at the same separation are very different; e.g. the x-space bulk two-point function diverges for lightlike separation whereas that of A_{LF} stays finite and fulfills the kind of lightlike locality known from 2-dim. conformal QFTs.

The reverse reconstruction (the inverse holography) of the physical fields from the LF fields goes again through algebraic steps; in that case one uses the generating property of the pointlike A_{LF} which together with (1) gives $\mathcal{A}(W)$ and reconstructs the system of wedge algebras for wedges in all possible positions by applying the Poincaré group. Finally from the net of wedge-localized algebras one obtains the *local algebraic structure of the bulk again via intersections.* There is no direct way to obtain to pass from the A_{LF} generating fields

[4] It can be shown that working with the x_+ derivates does not change the affiliated (weakly closed) operator algebras.

[5] That holographic pointlike generators are still related to the free fields in the bulk in a way which resembles a "restriction" is a speciality of the linear lightfront and does not extend to other null-surfaces. At least this aspect (5) of the old "lightcone quantization" was treated correctly.

to the bulk generating fields A rather the latter emerge as the local generators of these bulk (double cone) algebras for arbitrarily small localization regions. Clearly the possibility of directly constructing pointlike LF generators hinges on the mass-shell property of free fields; in the general interacting case the algebraic construction via intersections seems to be unavoidable and the problem of pointlike generators remains open, although the following generalization of the commutation relation to those of a *transversely extended chiral theory* has a high degree of plausibility

$$\left[B_{LF}^{(i)}(x_+,x_\perp),B_{LF}^{(k)}(x'_+,x'_\perp)\right]\simeq\left\{\sum_l\delta^{(n_l}(x_+-x'_+)B_{LF}^{(l)}(x_+,x_\perp)\right\}\delta(x_\perp-x'_\perp) \tag{6}$$

where in the case of transversely extended rational theories the algebraic structure of the theory permits a characterization in terms of a finite number of LF generating fields $B_{LF}^{(i)}$. For Wick monomials of free fields without derivatives this form of the relation can also be obtained directly by using their mass-shell representation for the individual fields in the Wick product. But In the interacting case one should expect that the algebraically determined net $\mathcal{A}(\mathcal{O})$ does not reproduce the original ∂W global algebra i.e. $\overline{\cup_{\mathcal{O}\subset\partial W}\mathcal{A}(\mathcal{O})}\subsetneq\mathcal{A}(\partial W)=\mathcal{A}(W)$. In field theoretic terms the bulk fields with a non-integer dimension are mapped into zero by algebraic holography since the bulk spin and the bulk short distance scale dimension have to match in order to pass the holographic lightfront projection. In this case there is the hope to recover $\mathcal{A}(\partial W)$ from its local net components by extending the net with the help of its superselection structure. There is also an indication that by using nonlocal formal representations of bulk Heisenberg fields in terms of incoming fields, one can define a more general holographic procedure which allows to incorporate bulk fields with anomalous short distance dimensions. The issue of holography applied to bulk fields with fractional short distance dimensions remains an important project for the future with the simplest case being that of two-dimensional factorizing models.

In the vast literature on "light-cone quantization" this problem as well as the question about the connection between the bulk- and the $LF-$ locality has not been properly addressed, and most of the computations about light-cone quantization use ad hoc prescriptions when interactions are present.

The appearance of the conformal invariant two-point functions in lightray direction (5) begs the question whether the two symmetry operations which the LF system of local algebras inherits from the bulk symmetries, namely the lightray translation and dilation (the restriction of the W-boost), can always be completed to the 3-parameter Moebius group. Using properties of the system of operator algebras $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset LF}$ one shows that a physically reasonable algebraic property[6], namely the non-triviality and cyclicity with respect to the vacuum state (the Reeh-Schlieder property of QFT[7] [8]) of the algebra $\mathcal{A}(\partial W_{a,b})$, guaranties the existence of a third one-parametric group of unitary operator $U(Rot)$

[6] This means one which can be checked for algebras of free fields and in whose formulation interaction plays no direct role.

[7] In the mathematical setting of operator algebras the cyclicity of an algebra and its com-

in the Hilbert space of the theory which leaves the vacuum invariant and (together with the lightray dilation and translation) generates the Moebius group [9].

At this point the operator-algebraic setting unfolds its "magic" in that new symmetries which were not present in the bulk theory arise in the holographic image from coherence properties of operator algebras in certain relative positions (modular inclusions) within a common Hilbert space. This symmetry-increase is typical for holographic projections onto null-surfaces and makes holographic projection a simplifrying and therefore useful tool. The above interaction-free case, which leads to an A_{LF} field which is similar to the 2-dim. abelian current algebra, admits a much larger unitarily implemented symmetry group, namely the diffeomorphism group of the circle. However the unitary implementers (beyond the Moebius group) do not leave the vacuum invariant and hence are not Wigner symmetries. These $\mathrm{Diff}(\mathrm{S}^1)$ may well appear in the holographic projection of interacting theories, even though the standard argument via a chiral energy-momentum tensor is not available for holography onto null-surfaces. As the vacuum-preserving chiral rotation results from algebraic properties of the holographic projection, it is conceivable that one finds algebraic properties which lead to the higher diffeomorphism symmetries. This would be very much in the spirit of the new setting of local covariance [10].

It is helpful to add one more remark with respect to the first statement in the above list. It is well-known that for spacelike separation in the bulk, even with a finite spacelike separation of two localization regions, there is no factorization on the vacuum vector Ω; in order to construct state vectors which lead to tensor-factorization one has to invoke the *split property* which is known to break down in the case that the two regions touch [8]. The holographic projection compresses the vacuum fluctuations into the lightlike direction and the relevant Hamitonian for the thermal manifestation (including localization-entropy) is the lightlike dilation generator which leaves ∂W invariant.

The second result in the above list follows from this tensor-factorization; transverse de-coupling implies additivity of extensive quantities. This is in particular the case for the entropy and energy caused by the vacuum fluctuations on the horizon; hence the contribution at the $x_+ = 0$ boundary of ∂W to the area density of the localization-entropy comes from vacuum fluctuations in lightray direction. Since in the limit of sharp localization we expect this density to become infinite, we introduce a variable finite *lightlike attenuation distance* ε for these vacuum polarization in order to study the limiting behavior for $\varepsilon \to 0$.

The computation of the leading ε-divergence is based on a theorem [1] which states that a chiral system in a vacuum state Ω, localized on a halfline $\mathbb{R}_+$, can be unitarily mapped to the full chiral algebra on $\mathbb{R}$ in a KMS thermal state $\Omega_{2\pi}$ at temperature 2π

$$(\mathcal{A}(\mathbb{R}_+), \Omega) \simeq (\mathcal{A}(\mathbb{R}), \Omega_{2\pi}) \tag{7}$$

with the unitary equivalence being given in terms of a conformal map which intertwines the dilation of the restricted system with the translation of the

mutant with respect to a vector state Ω is called the *standardness* property.

unrestricted (two-sided) thermal system. This "inverse Unruh effect" for chiral theories (i.e. a heat bath system is interpreted as a restricted vacuum system) is the key to the calculation of localization-entropy. Using the intuition from statistical mechanics we would conjecture that the entropy diverges proportional to the length i.e. as $l \cdot s_{2\pi}$ where $s_{2\pi}$ is the density per length. The intertwining map transforms the size l into $\varepsilon = e^{-l}$ so that the area density of the equivalent dilational system on the halfspace behaves as

$$s_{area} = |\ln \varepsilon|\, s_{2\pi} + finite, \quad \varepsilon \to 0 \tag{8}$$

The correctness of this idea can be checked by approximating the linear system by a sequence of finite systems using an "invariant" box approximation [1] in which the divergent partition function of the translative system is approximated by a rotational system in the limit of infinite temperature (interpreted as an infinite radius whose size is related to l) associated with the high temperature limit of the partition function associated to the Virasoro generator L_0. The calculation can be completed by the use of the chiral temperature duality for the partition function of $\hat{L}_0 = L_0 - \frac{c}{24}$ where the c is the Virasoro constant. It is precisely this shift in L_0 which gives the divergent l factor and identifies the constant $s_{2\pi}$ with $\frac{c}{6}$, so that the third result in the above list is obtained.

Chiral theories which originate from the chiral decomposition of conformal two-dimensional theories come with an energy momentum tensor which contains the Virasoro constant c, but, as mentioned before, this is not necessarily the case for Moebius covariant chiral theories which arise in holographic lightfront projection. The fact that it is precisely the vacuum shift in $\hat{L}_0$ which gives the expected l divergence in the length proportionality of the translative heat bath entropy may be taken as an indication that the Virasoro structure continues to hold for chiral theories from holographic projections. At the same time it gives an apparently new thermal interpretation to the Virasoro constant c in terms of properties of a lightlike thermal system. It is this inverse Unruh effect in combination with the simplification from the holographic projection which makes localization-entropy a useful concept.

It is an interesting question to what extend such entropy considerations apply to other null-surfaces including null-surfaces in Minkowski spacetime different from LF. Since the thermal phenomenon under consideration is caused by vacuum fluctuations near the boundary, one expects that the localization-entropy has an area behavior with the same leading ε-divergence as for the LF null-surface. This is corroborated by the structural theorem behind statement **1** (2) which secures the absence of transverse vacuum polarizations on null surfaces. It begs the further-going question whether there exist *pointlike generators* which obey commutations relations with transverse delta-functions similar to (5) or (6). Whereas for conformally invariant bulk theories one can rely on the global conformal equivalence of the wedge with a double cone, we have not been able to prove that this holographic behavior extends to massive theories on null-surfaces of double cones (the argument in [1] in favor of such a result was incorrect). If there exist pointlike generators on null-surfaces, they must lead to correlation

functions containing the quantum mechanical transverse delta-function factors because this is the only way to guaranty the absence of transverse vacuum fluctuations in pointlike generators.

Null-surfaces as those associated with double cones are quite interesting because one expects a quantum version of the classical Bondi-Metzner-Sachs symmetry in such a case. Whereas the quantum symmetries are always defined through their unitaries in the full Hilbert space (which only act geometrically on the horizon) which is the same as the one for the bulk, the BMS symmetry is only defined as an asymptotic transformation [11]. As a result of the more geometric nature (symmetry enhancement) of the action of the modular group restricted to the horizon as compared to its fuzzy action on massive bulk matter, one hopes to learn something about the modular action on the bulk. obtain a solution of unsolved problem of an analytic understanding of modular actions on localized bulk algebras. According to a recent proof of the Bisognano-Wichmann theorem by [12] based on a deep connection of modular theory with the Haag-Ruelle scattering theory, one can also expect to learn something about the action of the modular group on subwedge localized bulk algebras from its action in free theories. These problems of extensions of modular theory clearly go beyond the scope of an addendum and require a separate publication.

In the case of conformal models one can try to compute generators for double-cone holography by applying the relevant conformal transformation to wedge generators. The conformal map from the $x_0 - x_3$ wedge W to the radius=1 double cone $\mathcal{O}_1$ placed symmetrically around the origin is

$$\begin{aligned} &\mathcal{O}_1 = \rho(W + \frac{1}{2}e_3) - e_3,\ \rho(x) = -\frac{x}{x^2} \\ &W = \left\{(x_0, x_\perp, x_3) \mid x_3 > |x_0|, x_\perp \in R^2\right\} \end{aligned} \tag{9}$$

with e_3 being the unit vector in the 3-direction. Restricted to the (upper) horizon ∂W one obtains in terms of coordinates

$$\begin{aligned} &\partial\mathcal{O}_1 \ni (\tau, \vec{e}(1-\tau)),\ \tau = \frac{t}{t + x_\perp^2 + \frac{1}{4}},\ \vec{e} = \frac{1}{x_\perp^2 + \frac{1}{4}}(x_\perp, \frac{1}{4} - x_\perp^2) \\ &where\ \partial W = \left\{(t, x_\perp, t) \mid t > 0,\ x_\perp \in R^2\right\} \end{aligned} \tag{10}$$

If we use the unitary conformal transformation $\mathcal{A}(W) \rightarrow \mathcal{A}(\mathcal{O}_1)$ not only on global generators for $\partial\mathcal{A}(W) = \mathcal{A}(W)$ but also for their pointlike generationg fields A_{LF} (5), we obtain the desired compact transverse proportionality factor $\sim \delta(\vec{e} - \vec{e}')$ replacing $\delta(x_\perp - x_\perp')$ from the fact that the t-independent relation between $\vec{e}$ and $x_\perp$ is that of a stereographic projection of S^2 to R^2. The presence of this factor corroborates the absence of vacuum polarization in the above algebraic argument. The lightlike factor has the expected qualitative behavior in terms of the variable τ and the W-modular group $t \rightarrow e^\lambda t$ passes to the $\mathcal{O}_1$ modular automorphism

$$\tau \rightarrow \frac{-e^{-\lambda}(\tau + 1) + 1}{e^{-\lambda}(\tau + 1) + 1} \tag{11}$$

The transverse additive group passes via inverse stereographic transformation to the transverse rotational group. Again it is important to realize that the rotational symmetric two-point function of the pointlike generators of $\partial\mathcal{O}_1$ is not identical to the bulk two-point function restricted to $\partial\mathcal{O}_1$, in fact it is not even possible to obtain generators for $\mathcal{A}(\partial\mathcal{O}_1)$ by restricting pointlike bulk generators so that the only possibility consists in conformally transforming the lightfront generators (5).

The precise form of the lightray contribution to the local $\mathcal{O}_1$, including the proportionality factor for free fields, will be deferred to a future paper in which also the important problem of the applicability of ambient conformal maps restricted to null-surfaces of holographic projections $\partial W \to \partial\mathcal{O}_1$ in case of massive bulk theories will be investigated.

The divergence of localization-entropy in LQP is of course expected; since the it is related to a chiral heat bath situation by a conformal transformation which leads to a logarithmic distance parametrization which transforms the chiral "volume" factor L into the ε-attenuation distance factor $|\ln\varepsilon|$ for the vacuum polarization cloud. Hence the divergence is on the same structural-kinematical level as the large volume divergence of heat bath entropy i.e. it is not related to any still mysterious short distance divergencies caused by field generators in the bulk.

The important difference to previous global entropy calculations based on counting eigenstates of the standard time translation Hamitonian [13] is that in those calculations it is difficult to see the local origin of the phenomenon which is responsible for the area proportionality. There are general reasons which cast serious doubts on global calculations of vacuum entropy and energy which count the contribution from the occupation of global energy levels [14]. Such calculations tend to violate the recently formulated principle of local covariance of QFT in curved spacetime [10]. The present calculation does better on this issue since the Hamiltonian is the same as the one responsible for the Hawking-Unruh effect i.e. adapted to the invariance of the localization region and its horizon (i.e. the Lorentz boost generator in case of $\mathcal{A}(W)$ for generalizations to curved spacetime see next section). The theory is not modified by momentum space cutoffs or in any other conceptually and computationally uncontrollable way; one rather looks at a family of localization entropies for the holographically projected matter with a fuzzy boundary of a lightlike extension ε at the edge of the horizon. In particular this approach places into evidence that the rather universal behavior of vacuum polarization clouds in an attenuation distance ε near boundaries with thermal consequences should not to be blamed on the ultraviolet divergencies of particular pointlike fields (as they show up in renormalization theory), but is rather a physical consequence of the very peculiar operator-algebraic nature of causal localizability. Localized operator algebras are known to be monads i.e. von Neumann factor algebras of unique hyperfinite type III_1which have properties (e.g. no normal pure states) which easily escape the quantum mechanical intuition (which is limited to type I factor algebras [16]). Some of the problems black hole physics allegedly has with the foundations of QT may find their explanation in these peculiarities.

2 Extension to black hole horizons

It has been known for some time that QFT in CSTv with a bifurcated Killing horizon (bKh) lead to a similar situation as wedges and their horizons [15]. A presentation in the setting of operator algebras in curved space time for such a situation which turns out to be also suitable for the calculation of localization-entropy has been given in [4] following prior work in [17][18]. The main difference to the Minkowski situation is that at the start one only has a Killing symmetry and some assumptions about its geometrical action which guaranty the existence of a bKh. As a substitute for the vacuum state one needs a state vector Ω which is invariant under the action of the Killing symmetry; for the Schwarzschild-Kruskal black-hole spacetime this would be the Hartle-Hawking state [19]. In the analogy to the Minkowski spacetime the Killing symmetry corresponds to the wedge-preserving L-boost whose projection onto the bKh is the candidate for a dilation. It comes then as quite a surprise that it is possible to embed this Killing symmetry as a dilation into a full 3-parametric Moebius group, including a positive energy lightlike translation. In contradistinction to the LF holography these objects are only defined on the bKh; to be more precise they are implemented by operators in the Hilbert space of the QFT which leave Ω invariant; but they act geometrically only in their restriction to bKh. This is an illustration of the gain of symmetries through holographic projection. The definition of the net of algebras on the horizon is similar to the Minkowski case in that the wedge regions W possess a bKh analog; instead of the use of the Wigner little group to resolve the transverse locality structure on $\partial W_{a,b}$ one intersects $\mathcal{A}(\partial W_{a,b})$ with bulk algebras whose localization region contains the subregion of ∂W_{ab} of given transverse extension (tubular neighborhoods) to which one wants to localize the algebra. The only subtle part of the construction of the holographic projection to the horizon is the construction of the positive energy translation from the theory of modular inclusions. For this and other related details we refer to [4] which is the most authoritative account of this matter.

Having a horizon with a chiral lightray structure, the transverse factorization along the edge of the bKh and the area proportionality of entropy follows as in the lightfront case. There is no change in the divergent part of the area density; the so-called *surface gravity* which enters the relation between the dilation and the translation modifies only the next to leading (constant) term in the ε-expansion. Equating this entropy density with Bekenstein's classical area density would of course relate the size of the vacuum polarization cloud to the Planck length. The present purely structural approach cannot resolve the question whether different c-values belonging to a different quantum matter content really occur. But in case that the holographically projected matter does lead to different c-values this would lead to a problematization with the matter independent pure gravity-based Bekenstein density which is usually interpreted as including the contribution coming from quantum matter. In view of the fact that the state of black hole radiation of a collapsing star is not really an equilibrium state [20][21], one is of course entitled to doubt the relevance of localization-entropy or classical Bekenstein entropy and speculate that the

recent operator-algebraic concept based on an entropy flux [22] is more appropriate than the static entropy concept of equilibrium statistical mechanics.

As already remarked at the end of the previous section, the difference to the existing entropy calculations is conceptually significant. Whereas previous field theoretic calculations incorporate the finiteness of the Bekenstein law by introducing a global momentum space cutoff (which amounts to an uncontrollable modification of the theory), the present approach interprets the area behavior as a manifestation of absence of transverse vacuum fluctuations, leaving the numerical matching of the area densities to a better future understanding (or the key to QG) of how the back-reaction of quantum gravity determines the size ε of the halo of vacuum fluctuations at the edge of the horizon i.e. it is more in the spirit of a quasiclassical approximation which maintains the original QFT setting. Most of the astrophysical and cosmological vacuum effects (e.g. the cosmological constant) of recent interests have been calculated by quantum mechanical level occupation arguments i.e. by a procedure which does not comply with the principle of local covariance [14]. We believe that the present approach, based on holographic projection to null-surfaces, is exempt from this criticism.

Finally it is important to stress that the notion of holography which is in widespread use in the literature (originally introduced by 't Hooft) is different from the one in the present work. The main difference that the former is thought of as leading to a complete holographic image from which the full bulk theory can be recovered without any additional information about the bulk. Such an invertible "duality" relation between bulk and its holographic image, if possible at all, is expected by some people to arise "magically" from the still elusive quantum gravity. The present notion of holography onto null-surfaces applies to QFT in curved spacetime and lacks this uniqueness of inversion (unless one adds a few additional properties which refer to the ambient spacetime and enforce uniqueness). Even in the case of wedge localization it is not possible to reconstruct the localization structure within a wedge W from that of its horizon ∂W. But adding the knowledge about actions of Poincaré transformations outside the 7-parametric subgroup $\mathcal{G}$ of LF allows the reconstruction of the full net of wedge algebras and via intersection the full net of finitely localized bulk algebras and their possible generating pointlike fields. Since even in case of free fields there is no local connection between bulk fields and holographic field generators; the intervention of operator algebras is an essential aspect of holography. Apart from the structural results used in the present work, the operator algebra theory (in particular the Tomita-Takesaki modular theory) as one needs it the physical setting is still very much in its infancy; there is presently no good intuitive understanding of how geometric-physical properties in spacetime are related to the posititioning of hyperfinite type III_1 factor algebras in a common Hilbert space.

The relation of the conjectured quantum gravitational holography on null-surfaces to the present one is reminiscend to that of the conjectured Maldacena [23] gravity-gauge correspondence to Rehren's algebraic AdS-CFT correspondence [24]. The full invertibility (which justifies the use of the word correspondence) of the latter is the proven result of the exceptional fact that the causal

shadows cast by the regions of a "conformal brane" boundary at infinity of AdS can be intersected and subsequently transported anywhere (as a result of the maximal shared symmetry of bulk and boundary) to form the net on AdS; in the Maldacena setting AdS-CFT correspondence is interpreted as a conjectured result of the kind of quantum gravity inherent in string theory.

Acknowledgements: I thank my colleagues Joao Barata and Walter Wreszinski for their hospitality during a visit of the USP where part of this work was done. I also acknowledge the financial support of the FAPESP during a one year stay at the Mathematical Physics Group of the Physics Department of the USP.

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